# AIR BATTLES AND GROUND BATTLES--A COMMON PATTERN?

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## PREFACE

In the field of air-to-air combat, the state of the art has advanced to the point where models of one-versus-one air combat duels that compare well with test data can be developed. (For one example, see Reference 1.) Although models of larger-scale air battles have been devised (for one example see Reference 2), it has not been possible to put these models to the test of experience in any satisfyingly objective and quantitative manner. The problem of testing air battle models such as TAFCOM against the evidence of past battles is similar to that of testing complex land combat games. For the latter problem, two proposed methods have been put forward.

One of these suggests that the game be applied to "replay" some historical battle(s) to see whether or not the model satisfactorily "reproduces" the historical outcome(s). The usual difficulty with this method is that insufficient data are available on any one historical battle to carry out the proposed comparison. When sufficient data on a given battle can be found to pursue this approach, it is often found that the game is so complex that much of the data on the battle outcome must be supplied as an input to the game. Under these conditions, it is not at all easy or straightforward to determine how much of the agreement between the game and the historical battle is due to the validity of the model and how much is merely the result of having built the battle outcome into the game by assumption. What is more, the outcome data obtained from the specific historical battle and used as input to the

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game are usually (in fact, almost by definition) just those data which would not be known beforehand in applying the game to some other situation of interest. Thus, input data for the game might include the exact location and maneuver of forces on both sides, together with the amount of ammunition fired by each side and the manner in which this fire is allocated to the opposing targets. Clearly, in most battle situations, these kinds of information are known only in retrospect. If carefully done, a comparison of game output with historical outcomes may, of course, help to confirm the validity of those portions of the game which are not dependent on such input data. The point, however, is that the general usefulness of the game as a predictive tool remains heavily dependent on the quality of the input variables, and this is not checked by the comparison with a historical battle(s). Thus, for input data to the game as assumed above, the validity of game predictions would still depend absolutely on correct forecasts of the location and maneuver of forces on both sides, and on the amount and spatial distribution of fires on both sides. Since these are among the more difficult types of data to accurately forecast, the utility of the game as an analytical device remains in limbo even if game output and historical outcome are found to be generally similar.

The second serious approach to testing combat models against historical data is more subtle and indirect. It has been described in detail elsewhere (see References 3, 4, 5, and 6) and so will only be outlined here. The basic idea of the procedure is to find regularities or "patterns" in historical battle data, and then to determine whether or not the model exhibits a similar "pattern." This approach seems to be of a more fundamental nature than that of simply comparing individual cases. As such, the extrapolation of the model to new types of situations may be made with more confidence if it has been validated using the second method. Essentially, no data on the historical battles needs to be used to supply inputs to the model under test, thus maintaining a clear separation between the model and the historical data against which the model output is to be compared. Since the method depends on the "pattern" exhibited in historical battle data rather than on the outcome of any individual battle, data inaccuracies and absences in

individual battles need not seriously affect the results, provided only that they tend to average each other out in a large collection of battles. The obvious drawback is the prerequisite that a fairly well-established historical "pattern" be known.

To supply this necessary item, in this paper we propose a pattern of behavior that seems to be exhibited by historical air battles. This pattern is developed by analyzing data on the Battle of Britain as given in Reference 7, and comparing the results obtained to similar findings already known to hold for the ground combat situation (see, for example, References 9, 10, 11, 12, and 13). The fact that this pattern is shared both by land battles and by the Battle of Britain (where the action was fought entirely in the air) suggests that it may be a fundamental characteristic of all military combat situations. It would be desirable to continue investigating this possibility by collecting and analyzing additional battle data.

The results presented here obviously are of intrinsic value independent of any application of the findings to testing war games or other models of combat, and should be of interest to all students of the military art.

## SUMMARY

Let us write Lanchester's attrition equations in the following form:

$$\dot{x} = - Dy$$

$$\dot{y} = - Ax$$

where $x = x(t)$ is the number of surviving elements on the attacker's side and $y = y(t)$ is the number of surviving elements on the defender's side as a function of the time, $t$, after the start of the engagement. The dot indicates differentiation with respect to $t$. The parameters $A$ and $D$ will be called the activity parameters and their ratio, $D/A$, the activity ratio. If the forces on both sides at the start and

finish of an engagement are known, we may estimate the value of the activity ratio for that engagement by the formula:

$$D/A = (x_0^2 - x^2)/(y_0^2 - y^2)$$

where $x_0$ and $y_0$ are the initial strengths of the two forces and x and y are their terminal strengths. A relation between the activity ratio and the initial force ratio, $x_0/y_0$ is known to hold for historical land battles. Using data on the Battle of Britain, it is shown that this air battle obeys the same relation between activity ratio and initial force ratio. Is it possible that this same pattern of dependency of activity ratio on force ratio is a general characteristic of all armed combat? On the basis of the data now in hand, it is at least a reasonable hypothesis which should be tested further by the collection and analysis of additional data. Should such efforts result in supporting the observed pattern's general applicability to combat situations, this would be a finding of great importance to our understanding of combat dynamics.

## BACKGROUND

We begin by writing Lanchester's equations in the following form:

$$\dot{x} = - Dy$$
$$\dot{y} = - Ax$$

where $x = x(t)$ is the number of surviving elements on the attacker's side and $y = y(t)$ is the number of surviving elements on the defender's side as a function of the time, t, after the start of the engagement. The dot indicates differentiation with respect to t. The parameter A gives the rate at which the defending elements are eliminated per attacking element per unit time. Similarly, D gives the rate of attrition to the attacker expressed as attack elements lost per defender element per unit time. The parameters A and D will be called the activity parameters.

Dividing the first equation by the second, we can write

$$x(dx) = (D/A)\, y(dy)$$

which can immediately be integrated to yield

$$D/A = (x_0^2 - x^2)/(y_0^2 - y^2)$$

where $x_0$ and $y_0$ are the initial strengths of the attacker and defender.

By putting

$$a = x(t)/x_0$$

$$d = y(t)/y_0$$

where a and d are the surviving fraction of attacker and defender strength as a function of time t into the battle, we may write the original equations as:

$$\dot{a} = -\delta d$$

$$\dot{d} = -\alpha a$$

where

$$\alpha = A(x_0/y_0)$$

$$\delta = D(y_0/x_0)$$

are new "normalized" activity parameters. Their physical meaning can be found by considering that, at the initiation of the action, $\dot{a} = \dot{a}(0)$ and $d = d(0) = 1$, so that by the relation given above for $\dot{a}$,

$$\dot{a}(0) = -\delta$$

similarly,

$$\dot{d}(0) = -\alpha\ .$$

Consequently, $\alpha$ and $\delta$ should be interpreted as the defender's and attacker's initial fractional attrition rates, expressed in units such as percent per unit time.

Dividing the equations for $\dot{a}$ and $\dot{d}$ and solving as before yields

$$\begin{aligned}\delta/\alpha &= (1 - a^2)/(1 - d^2) \\ &= \mu^2\end{aligned}$$

where we have introduced the new symbol $\mu^2$ to stand for $\delta/\alpha$.

In this form we see that $\mu$ is related to the relative advantage of the two sides. In particular, if $\mu > 1$, then the attacker's surviving fraction goes to zero before the defender's does; if $\mu < 1$, then the defender's surviving fraction is the first to reach zero; if $\mu = 1$, then the attacker's and defender's surviving fractions both reach zero at the same time. This suggests that we define a defender's "advantage parameter," V, by the relation:

$$V = \ln(\mu) \ ,$$

where "ln" denotes "natural logarithm of." Then V will range from $-\infty$ to $+\infty$, and will be $> 0$ if the defender has the advantage, $< 0$ if the attacker has the advantage, and $= 0$ is neither side has the advantage.

The parameters we have introduced so far are interrelated as follows:

$$\begin{aligned}\mu^2 &= (1 - a^2)/(1 - d^2) \\ &= \delta/\alpha \\ &= (D/A)(y_0/x_0)^2 \\ &= \dot{a}(0)/\dot{d}(0)\end{aligned}$$

The complete solution of the differential equation for a and d subject to the initial conditions $a(0) = 1$, $d(0) = 1$ can be written as:

$$a = \cosh \epsilon - \mu \sinh \epsilon$$

$$d = \cosh \epsilon - \mu^{-1} \sinh \epsilon$$

where $\mu$ is as previously defined and $\epsilon = \lambda t$. Here $\lambda = \sqrt{\alpha\delta}$ and t is the time into the engagement. Referring to the definition of $\alpha$ and $\delta$, we see that

$$\lambda = \sqrt{\alpha\delta}$$

$$= \sqrt{AD} \quad ,$$

i.e., $\lambda$ is the geometric mean of the attacker's and defender's activity parameters. Since it represents an average activity, it seems appropriate to call $\lambda$ the "intensity" parameter and to interpret it as a measure of the average intensity of the battle. With this interpretation of $\lambda$, consider

$$\epsilon = \lambda T$$

where T is the total time duration of the battle. Since this $\epsilon$ is the product of battle duration and average battle intensity, it may be called the "bitterness" parameter and interpreted as a measure of the bloodiness of the battle. It is possible to determine an empirical value for $\epsilon$ even if the battle duration is not known. The appropriate equation (see Reference 10) is

$$\epsilon = \ln((1 + \mu)/(a + d\mu)) \; .$$

If the historical record includes the values of $x_0$, $y_0$, x, and y for some battle, then it is possible to estimate the following Lanchester parameters for the battle:

- o Force Ratio $(x_0/y_0)$
- o Surviving Fractions (a and d)
- o Advantage (V or $\mu$)

- Activity Ratio (D/A)
- Bitterness ($\epsilon$)

For land combat engagement, several studies of these parameters have been performed and have led to the statement of certain patterns or generalizations that summarize the results of these investigations. Here we use the recapitulation of findings presented in Reference 10 to obtain the following general statements:

- The Lanchester parameters defined above appear to be valid indices of the real-world phenomena identified by the names assigned to the various parameters.
- Logarithmic bitterness, $\ln\epsilon$, is approximately normally distributed with mean -2.16 and standard deviation 0.83.
- Logarithmic activity ratio, ln(D/A), is linearly correlated with logarithmic force ratio, $\ln (x_0/y_0)$. The relation is given by

$$\ln (D/A) = 0.230 + 1.266 \ln (x_0/y_0).$$

The corresponding correlation coefficient is 0.603. The standard deviation of estimate for the regression is 0.594, and the standard deviation of the estimated slope is 0.122. These statistics are based on a sample of 92 historical land combat battles, most of which took place during the 19th century.

## BATTLE OF BRITAIN DATA ANALYSIS

We wish to compare the results just summarized for historical land battles with data for the Battle of Britain. We first present the Battle of Britain data that will be used for this purpose.

References 7 and 8 were consulted for applicable data. However, Mason (Reference 8) gives only data on losses to each side with no information on the number of sorties flown by either side. Collier

(Reference 7) gives losses to both sides, and usually gives the number of sorties flown by the British Fighter Command. However, he only occasionally gives the sorties flown by the German Air Force. There are two periods where Collier gives daily sorties and losses on both sides. These are the period from 13 August 1940 to 16 August 1940, inclusive, and the period from 24 August 1940 to 6 September 1940, inclusive. Collier identifies the losses involved simply as "losses," and they presumably include operational attrition as well as combat losses. Although they may also include the losses caused by anti-aircraft artillery, it seems that the losses directly attributable to anti-aircraft weapons are only a relatively small fraction of the total and so are a minor factor. The British anti-aircraft weapons apparently served mostly to break up attacking German Air Force formations and thus reduced their capacity for mutual support. Although Collier occasionally presents data separately for sorties flown during the day and those flown during the night, he gives losses only for an entire 24-hour interval. Accordingly, there is no objective way to allocate the reported losses to day or night operations. Instead, the sorties will be combined to give the total number flown in a 24-hour interval. With these conventions, and treating the German Air Force as being in the role of the "attacker," the Collier sortie and loss data are as reproduced in Table I. (If Mason's loss data are used in place of Collier's, the results obtained in the sequel are not materially different.)

The Lanchester parameters developed on the basis of the data of Table I are presented in Table II. We are now in a position to compare the pattern exhibited by these Lanchester parameters with those mentioned earlier for the historical land combat battles. We begin by comparing the distribution of bitterness values. The land combat values have a normally-distributed logarithmic bitterness ($\ln \epsilon$) with mean -2.16 and standard deviation 0.83. For the Battle of Britain, the logarithmic bitterness has a mean value of -3.65 and a sample standard deviation of 0.30. The highest logarithmic bitterness value for the Battle of Britain data is -3.24, or 1.3 standard deviations below the land battle mean logarithmic bitterness. This means that <u>all</u> of the Battle of Britain data have logarithmic bitterness values below the

Table I

COLLIER'S DATA FOR THE BATTLE OF BRITAIN

| No. | GAF Attack Sorties, $x_0$ | Fighter Command Defense Sorties, $y_0$ | GAF Losses, $c_x$ | Fighter Command Losses, $c_y$ | Date |
|---|---|---|---|---|---|
| 1 | 1485 | 700 | 45 | 13 | 8-13-40 |
| 2 | 489 | 494 | 19 | 8 | 8-14-40 |
| 3 | 1786 | 974 | 75 | 34 | 8-15-40 |
| 4 | 1715 | 776 | 45 | 21 | 8-16-40 |
| 5 | 1200 | 981 | 38 | 22 | 8-24-40 |
| 6 | 880 | 524 | 20 | 16 | 8-25-40 |
| 7 | 1088 | 829 | 41 | 31 | 8-26-40 |
| 8 | 225 | 335 | 9 | 1 | 8-27-40 |
| 9 | 976 | 761 | 30 | 20 | 8-28-40 |
| 10 | 940 | 526 | 17 | 9 | 8-29-40 |
| 11 | 1605 | 1054 | 36 | 26 | 8-30-40 |
| 12 | 1620 | 1007 | 41 | 39 | 8-31-40 |
| 13 | 820 | 690 | 14 | 15 | 9-1-40 |
| 14 | 1047 | 780 | 35 | 31 | 9-2-40 |
| 15 | 676 | 745 | 16 | 16 | 9-3-40 |
| 16 | 947 | 698 | 25 | 17 | 9-4-40 |
| 17 | 903 | 712 | 23 | 20 | 9-5-40 |
| 18 | 797 | 1031 | 35 | 23 | 9-6-40 |
| COMPOSITE | 19199 | 13617 | 564 | 362 | |

Table II

LANCHESTER PARAMETERS FOR THE BATTLE OF BRITAIN

| No. | Date | $x_0/y_0$ | a | d | $\mu$ | V | D/A | $\epsilon$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 8-13-40 | 2.121 | 0.970 | 0.981 | 1.274 | 0.242 | 7.300 | 0.024 |
| 2 | 8-14-40 | 0.990 | 0.961 | 0.984 | 1.540 | 0.432 | 2.324 | 0.025 |
| 3 | 8-15-40 | 1.834 | 0.958 | 0.965 | 1.095 | 0.091 | 4.030 | 0.039 |
| 4 | 8-16-40 | 2.210 | 0.974 | 0.973 | 0.985 | -0.015 | 4.738 | 0.027 |
| 5 | 8-24-40 | 1.223 | 0.968 | 0.978 | 1.186 | 0.170 | 2.103 | 0.027 |
| 6 | 8-25-40 | 1.679 | 0.977 | 0.969 | 0.864 | -0.146 | 2.108 | 0.027 |
| 7 | 8-26-40 | 1.312 | 0.962 | 0.963 | 1.004 | 0.004 | 1.736 | 0.038 |
| 8 | 8-27-40 | 0.672 | 0.960 | 0.997 | 3.626 | 1.288 | 5.933 | 0.011 |
| 9 | 8-28-40 | 1.283 | 0.969 | 0.974 | 1.080 | 0.077 | 1.919 | 0.029 |
| 10 | 8-29-40 | 1.787 | 0.982 | 0.983 | 1.028 | 0.027 | 3.374 | 0.018 |
| 11 | 8-30-40 | 1.523 | 0.978 | 0.975 | 0.954 | -0.047 | 2.111 | 0.024 |
| 12 | 8-31-40 | 1.609 | 0.975 | 0.961 | 0.811 | -0.209 | 1.703 | 0.032 |
| 13 | 9-1-40 | 1.188 | 0.983 | 0.978 | 0.887 | -0.120 | 1.112 | 0.019 |
| 14 | 9-2-40 | 1.342 | 0.967 | 0.960 | 0.919 | -0.085 | 1.520 | 0.037 |
| 15 | 9-3-40 | 0.907 | 0.976 | 0.979 | 1.049 | 0.048 | 0.906 | 0.023 |
| 16 | 9-4-40 | 1.357 | 0.974 | 0.976 | 1.041 | 0.040 | 1.993 | 0.026 |
| 17 | 9-5-40 | 1.268 | 0.975 | 0.972 | 0.953 | -0.048 | 1.460 | 0.027 |
| 18 | 9-6-40 | 0.773 | 0.956 | 0.978 | 1.395 | 0.333 | 1.164 | 0.032 |
| COMPOSITE | | 1.410 | 0.971 | 0.973 | 1.051 | 0.049 | 2.194 | 0.028 |

value that would occur by chance alone 10 percent of the time were they distributed according to the same pattern as are the historical land battle data. For this to have happened 18 times in succession is unlikely. Consequently, we must conclude that the Battle of Britain data represent battles with a significantly lower bitterness than is exhibited in the historical land battles.

Next we turn our attention to the relationship between activity ratio and force ratio. The results are most conveniently presented in graphical form as shown in Figure 1. Here we plot the Battle of Britain Lanchester parameters D/A against $x_0/y_0$ on log-log graph paper. Also shown are the mean regression lines obtained from an analysis of the historical land battles and $\pm$ 1.5 standard deviation contour lines about the mean regression line. These contour lines can be expected to include all but about 13 percent of the data points if the Battle of Britain data exhibit the same pattern as the historical land battles. Indeed, with the exception of but one point (number 8, of 8-27-40) the agreement between the Battle of Britain data and that for the historical land combat data is striking. Inspection of Table I indicates that the exceptional point has very low losses compared with the other points plotted. Had the losses been 4 and 5 instead of 9 and 1 (or even if they had been 5 and 4), the resulting activity ratio value would have plotted within the $\pm$ 1.5 standard deviation contour band.

Rejecting the one exceptional point (number 8, of 8-27-40), and computing the regression statistics for the remaining 17 points, yields the values shown in Table III as compared to those for the historical land battles. The degree of agreement between the regression statistics for the Battle of Britain and those for the historical land battles is encouraging. Considering the vast difference between land and air battles with respect to the circumstances and manner in which they are conducted, the degree of agreement actually exhibited by the data is rather unexpected. Is it possible that this same pattern of dependency of activity ratio on force ratio is a general characteristic of all armed combat? On the basis of the data now in hand, it is at least a reasonable hypothesis which should be tested further by the collection and analysis of additional data. Should such efforts result

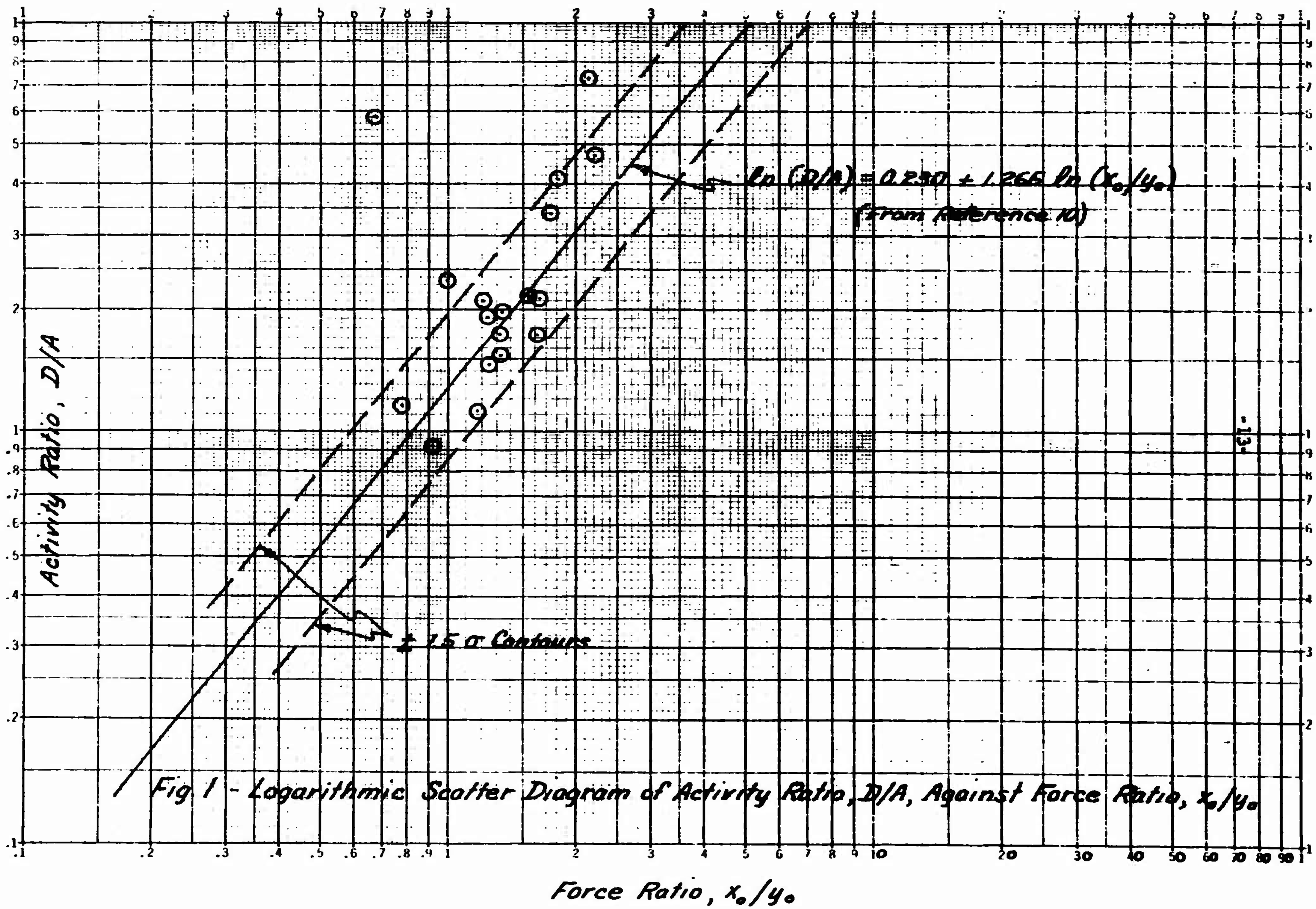


Fig 1 - Logarithmic Scatter Diagram of Activity Ratio, D/A, Against Force Ratio, $x_o/y_o$

Table III

COMPARATIVE REGRESSION STATISTICS OF $\ln(D/A)$ on $\ln(x_0/y_0)$

| Statistic | Battle of Britain | Historical Land Battles |
|---|---|---|
| Regression line intercept | 0.242 | 0.230 |
| Regression line slope | 1.544 | 1.266 |
| Correlation coefficient | 0.729 | 0.603 |
| Standard deviation about the mean regression line | 0.322 | 0.594 |
| Standard deviation of regression line slope | 0.282 | 0.122 |

in supporting the observed pattern's general applicability to combat situations, this would be a finding of great importance to the understanding of combat dynamics.

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